320_1.pdf

LAST NAME:

FIRST NAME: Solution

THEORY OF COMPUTATION
CSCI 320, course # 44287
TEST # 1
March 19, 2014
instructor: Bojana Obrenić

NOTE: It is the policy of the Computer Science Department to issue a failing grade in the course to any student who either gives or receives help on any test.

Your ability and readiness to follow the test protocol described below is a component of the technical proficiency evaluated by this test. If you violate the test protocol you will thereby indicate that you are not qualified to pass the test.

this is a **closed-book** test, to which it is **forbidden** to bring anything that functions as: paper, calculator, hand-held organizer, computer, telephone, voice or video recorder or player, or any device other than pencils (pens), erasers and clocks;

answers should be written only in the space marked "**Answer:** " that follows the statement of the problem (unless stated otherwise);

scratch should never be written in the answer space, but may be written in the enclosed scratch pad, the content of which *will not be graded;*

any problem to which you give **two or more (different) answers** receives the **grade of zero** automatically;

student name has to be written **clearly** on **each page** of the problem set and on the first page of **scratch pad** the during the **first five minutes of the test**—there is a penalty of **at least 1 point** for each missing name;

when requested, **hand in** the problem set together with the scratch pad;

once you leave the classroom, you cannot come back to the test;

your **handwriting** must be legible, so as to leave no ambiguity whatsoever as to what exactly you have written.

You may work on as many (or as few) problems as you wish.
time: 75 minutes.

each **fully** solved problem: 16 points.
full credit: 80 points.
C: 44 points.

Good luck.

problem:	01	02	03	04	05	06	07	total: [%]
grade:								

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Problem 1 Let: $\Sigma = \{a, b, c, d\}$ and let L be the language defined by the regular expression:

$$(a \cup b \cup c)\,(b \cup c \cup d)\,(\lambda \cup a)$$

State the cardinality of each of the following sets. (For a finite set, state the exact number. For an infinite set, state that it is infinite and specify whether it is countable or not.)

Note: If you are to receive any credit for this problem, the number of your correct answers must exceed the number of incorrect ones. In other words, every incorrect answer cancels out the credit earned by one correct answer; a missing answer is neither correct nor incorrect. If the score on this problem is negative, a score of zero is assigned for this problem.

1. set of all strings over Σ with length equal to 3

   Answer: 64

2. set of all subsets of Σ with exactly 3 elements

   Answer: 4

3. set of all languages over Σ that contain exactly 3 strings

   Answer: infinite and countable

4. set of all strings in L whose length is equal to 3

   Answer: 9

5. set of all regular expressions over Σ

   Answer: infinite and countable

6. set of all context-free grammars over Σ

   Answer: infinite and countable

7. set of all finite subsets of Σ^*

   Answer: infinite and countable

8. set of all infinite subsets of Σ^*

   Answer: infinite and uncountable

9. L

   Answer: 18

10. L^*

    Answer: infinite and countable

11. $\overline{L}$ (complement of L in Σ^*)

    Answer: infinite and countable

~~12.~~ set whose regular expression is $\lambda \cup \emptyset \cup a$

Answer: || 3 2

not.

13. set whose regular expression is $\lambda^*\ \emptyset^*\ a\,b$

    Answer: 1

14. set whose regular expression is $\lambda\ \emptyset\ a$

    Answer: 0

15. set whose regular expression is $\lambda^* \cup \emptyset^* \cup a \cup b$

    Answer: 3

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Problem 2 Let L_1 be the language represented by the regular expression:

$$a^*d^*b^*c^*$$

Let L_2 be the language generated by the context-free grammar $G_2 = (V, \Sigma, P, S_2)$, where $\Sigma = \{a, b, c, d\}$, $V = \{S_2\}$, and the production set P is:

$$S_2 \to aS_2cc \mid ddS_2b \mid \lambda$$

(a) Write 6 distinct strings that belong to L_1 but do not belong to L_2 (belong to $L_1 \setminus L_2$). If such strings do not exist, state it and explain why.

Answer:

a, d, b, c, ab, ac

(b) Write 6 distinct strings that belong to L_2 but do not belong to L_1 (belong to $L_2 \setminus L_1$). If such strings do not exist, state it and explain why.

Answer:

ddaccb, ddaaccccb,

ddddaccbb, ddddaaccccbb,

ddddddaaaccccccbbb,

dddddddaccbbbb

(c) Write 6 distinct strings that belong to L_1 and L_2 (belong to $L_1 \cap L_2$). If such strings do not exist, state it and explain why.

Answer:

λ, acc, ddb, addbcc, addddbbcc,

aaddbcccc

(d) Write 6 distinct strings over alphabet $\{a, b, c, d\}$ that do not belong to L_1 and do not belong to L_2 (belong to $\overline{L_1} \cap \overline{L_2}$). If such strings do not exist, state it and explain why.

Answer:

ca, ba, da, bd, cd, bdd

(e) Write 6 distinct strings that belong to L_2^* but do not belong to L_2. If such strings do not exist, state it and explain why.

Answer:

accacc, ddbddb, accddb,

ddbacc, addbccacc, ddaccbddb

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Problem 2 Let L_1 be the language represented by the regular expression:

$$b^*d^*a^*c^*$$

Let L_2 be the language generated by the context-free grammar $G_2 = (V, \Sigma, P, S_2)$, where $\Sigma = \{a, b, c, d\}$, $V = \{S_2\}$, and the production set P is:

$$S_2 \rightarrow bS_2cc \mid ddS_2a \mid \lambda$$

(a) Write 6 distinct strings that belong to L_1 but do not belong to L_2 (belong to $L_1 \setminus L_2$). If such strings do not exist, state it and explain why.

Answer:

1. bdac
2. bdacc
3. bbdacc
4. bbbdacc
5. a
6. c

(b) Write 6 distinct strings that belong to L_2 but do not belong to L_1 (belong to $L_2 \setminus L_1$). If such strings do not exist, state it and explain why.

Answer:

1. ddbcca
2. ddbbcccca
3. ddbbbcccccca
4. ddddbccaa
5. ddddddbccaaa
6. ddddbbccccaa

(c) Write 6 distinct strings that belong to L_1 and L_2 (belong to $L_1 \cap L_2$). If such strings do not exist, state it and explain why.

Answer:

1. λ
2. bcc
3. dda
4. bddacc
5. bbcccc
6. ddddaa

(d) Write 6 distinct strings over alphabet $\{a, b, c, d\}$ that do not belong to L_1 and do not belong to L_2 (belong to $\overline{L_1} \cap \overline{L_2}$). If such strings do not exist, state it and explain why.

Answer:

1. cb
2. ca
3. cd
4. ad
5. ab
6. db

(e) Write 6 distinct strings that belong to L_2^* but do not belong to L_2. If such strings do not exist, state it and explain why.

Answer:

1. bccbcc
2. ddadda
3. bddaccbddacc
4. ddbccaddbcca
5. bbccccbbcccc
6. ddddaaddddaa

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Problem 2 Let L_1 be the language represented by the regular expression:

$$d^* a^* b^* c^*$$

Let L_2 be the language generated by the context-free grammar $G_2 = (V, \Sigma, P, S_2)$, where $\Sigma = \{a, b, c, d\}$, $V = \{S_2\}$, and the production set P is:

$$S_2 \to ddS_2c \mid aS_2bb \mid \lambda$$

(a) Write 6 distinct strings that belong to L_1 but do not belong to L_2 (belong to $L_1 \setminus L_2$). If such strings do not exist, state it and explain why.

Answer:

d
a
b
c
db
ac

(b) Write 6 distinct strings that belong to L_2 but do not belong to L_1 (belong to $L_2 \setminus L_1$). If such strings do not exist, state it and explain why.

Answer:

addcbb
addddccbb
addddddcccbb
addddddddccccbb
addddddddddcccccbb
addddddddddddccccccbb

(c) Write 6 distinct strings that belong to L_1 and L_2 (belong to $L_1 \cap L_2$). If such strings do not exist, state it and explain why.

Answer:

ddc
abb
ddabbc
λ
ddaabbbbc
ddaaabbbbbbc

(d) Write 6 distinct strings over alphabet $\{a, b, c, d\}$ that do not belong to L_1 and do not belong to L_2 (belong to $\overline{L_1} \cap \overline{L_2}$). If such strings do not exist, state it and explain why.

Answer:

ad
bd
cd
ca
cb
ba

(e) Write 6 distinct strings that belong to L_2^* but do not belong to L_2. If such strings do not exist, state it and explain why.

Answer:

ddcddc
abbabb
addcbbaddcbb
ddcddcddc
abbabbabb
abbabbabbabb

Problem 2 Let L_1 be the language represented by the regular expression:

$$c^*d^*b^*a^*$$

Let L_2 be the language generated by the context-free grammar $G_2 = (V, \Sigma, P, S_2)$, where $\Sigma = \{a, b, c, d\}$, $V = \{S_2\}$, and the production set P is:

$$S_2 \to ccS_2a \mid dS_2bb \mid \lambda$$

(a) Write 6 distinct strings that belong to L_1 but do not belong to L_2 (belong to $L_1 \setminus L_2$). If such strings do not exist, state it and explain why.

Answer: 1) c
2) cd
3) d
4) cdbb
5) cdd
6) cddd

(b) Write 6 distinct strings that belong to L_2 but do not belong to L_1 (belong to $L_2 \setminus L_1$). If such strings do not exist, state it and explain why.

Answer: 1) dccabb
2) dccccaabb
3) dccccccaaabb
4) dccccccccaaaabb
5) dccccccccccaaaaabb
6) dccccccccccccaaaaaabb

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(c) Write 6 distinct strings that belong to L_1 and L_2 (belong to $L_1 \cap L_2$). If such strings do not exist, state it and explain why.

Answer: 1) ccdbba
2) λ
3) ccddbbbba
4) ccdddbbbbbba
5) ccddddbbbbbbbba
6) dbb

(d) Write 6 distinct strings over alphabet $\{a, b, c, d\}$ that do not belong to L_1 and do not belong to L_2 (belong to $\overline{L_1} \cap \overline{L_2}$). If such strings do not exist, state it and explain why.

Answer: 1) cad
2) dab
3) bad
4) ad
5) ac
6) ab

(e) Write 6 distinct strings that belong to L_2^* but do not belong to L_2. If such strings do not exist, state it and explain why.

Answer: 1) ccadbb
2) ccacca
3) dbbdbb
4) ccaccadbb
5) ccadbbcca
6) dbbccadbb

Problem 2 Let L_1 be the language represented by the regular expression:

$$c^*d^*b^*a^*$$

Let L_2 be the language generated by the context-free grammar $G_2 = (V, \Sigma, P, S_2)$, where $\Sigma = \{a, b, c, d\}$, $V = \{S_2\}$, and the production set P is:

$$S_2 \to ccS_2a \mid dS_2bb \mid \lambda$$

(a) Write 6 distinct strings that belong to L_1 but do not belong to L_2 (belong to $L_1 \setminus L_2$). If such strings do not exist, state it and explain why.

Answer:

a, b, c, d, ad, ab

(b) Write 6 distinct strings that belong to L_2 but do not belong to L_1 (belong to $L_2 \setminus L_1$). If such strings do not exist, state it and explain why.

Answer:

dccabb
dccccaabb
ddccabbbb
ddccccaabbbb
dddccabbbbbb
dddccccaabbbbbb

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(c) Write 6 distinct strings that belong to L_1 and L_2 (belong to $L_1 \cap L_2$). If such strings do not exist, state it and explain why.

Answer:

cca
dbb
ccdbba
ccccaa
ddbbbb
~~c~~cccdbbaa

(d) Write 6 distinct strings over alphabet $\{a, b, c, d\}$ that do not belong to L_1 and do not belong to L_2 (belong to $\overline{L_1} \cap \overline{L_2}$). If such strings do not exist, state it and explain why.

Answer:

ab
ac
ad
abc
abd
abcd

(e) Write 6 distinct strings that belong to L_2^* but do not belong to L_2. If such strings do not exist, state it and explain why.

Answer:

ccadbb,
dbbcca
ccadbbcca
dbbccadbb
ccadbbdbb
dbbccacca

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Problem 2 Let L_1 be the language represented by the regular expression:

$$d^*a^*b^*c^*$$

Let L_2 be the language generated by the context-free grammar $G_2 = (V, \Sigma, P, S_2)$, where $\Sigma = \{a, b, c, d\}$, $V = \{S_2\}$, and the production set P is:

$$S_2 \to ddS_2c \mid aS_2bb \mid \lambda$$

(a) Write 6 distinct strings that belong to L_1 but do not belong to L_2 (belong to $L_1 \setminus L_2$). If such strings do not exist, state it and explain why.

Answer:

dabc

bc

d

a

b

c

— x

(b) Write 6 distinct strings that belong to L_2 but do not belong to L_1 (belong to $L_2 \setminus L_1$). If such strings do not exist, state it and explain why.

Answer:

x $\lambda \in L_1$ No.

addcbb

aaddcbbbb

aaaddcbbbbbb

aaaaddcbbbbbbbb

aaaaaddcbbbbbbbbbb

(c) Write 6 distinct strings that belong to L_1 and L_2 (belong to $L_1 \cap L_2$). If such strings do not exist, state it and explain why.

Answer:

ddc

abb

ddddcc

aabbbb

ddddddccc

aaabbbbbb

(d) Write 6 distinct strings over alphabet $\{a, b, c, d\}$ that do not belong to L_1 and do not belong to L_2 (belong to $\overline{L_1} \cap \overline{L_2}$). If such strings do not exist, state it and explain why.

Answer:

ad

cd

bd

bcd

bcad

cabd

(e) Write 6 distinct strings that belong to L_2^* but do not belong to L_2. If such strings do not exist, state it and explain why.

Answer:

ddcddc

abbabb

ddcddcddc

abbabbabb

ddcddcddcddc

abbabbabbabb

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Problem 2 Let L_1 be the language represented by the regular expression:

$$a^*d^*b^*c^*$$

Let L_2 be the language generated by the context-free grammar $G_2 = (V, \Sigma, P, S_2)$, where $\Sigma = \{a, b, c, d\}$, $V = \{S_2\}$, and the production set P is:

$$S_2 \to aS_2cc \mid ddS_2b \mid \lambda$$

(a) Write 6 distinct strings that belong to L_1 but do not belong to L_2 (belong to $L_1 \setminus L_2$). If such strings do not exist, state it and explain why.

Answer:

ac, a, cc, aaa, ad, db

(b) Write 6 distinct strings that belong to L_2 but do not belong to L_1 (belong to $L_2 \setminus L_1$). If such strings do not exist, state it and explain why.

Answer:

addaccbcc,

addaddbccbcc,

addaddaccbccbcc,

addaddaddaccbccbccbcc,

addaddaddaddaccbccbccbccbcc

a dd add add add addacc bccbcc bcc bcc bcc

(c) Write 6 distinct strings that belong to L_1 and L_2 (belong to $L_1 \cap L_2$). If such strings do not exist, state it and explain why.

Answer:

addbcc,

acc,

* ||| dbb, $\boxed{ddb}$

bo.

addbcc

aacccc

aaacccccc

(d) Write 6 distinct strings over alphabet $\{a, b, c, d\}$ that do not belong to L_1 and do not belong to L_2 (belong to $\overline{L_1} \cap \overline{L_2}$). If such strings do not exist, state it and explain why.

Answer:

dbca

ccca

ba

ca

dda

bd

(e) Write 6 distinct strings that belong to L_2^* but do not belong to L_2. If such strings do not exist, state it and explain why.

Answer:

addacc bcc addaccbcc

acc acc acc

ddb ddb

ddb ddb ddb

aacccc aacccc

dd ddb b dd ddbb

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Problem 2 Let L_1 be the language represented by the regular expression:

$$c^*d^*b^*a^*$$

Let L_2 be the language generated by the context-free grammar $G_2 = (V, \Sigma, P, S_2)$, where $\Sigma = \{a, b, c, d\}$, $V = \{S_2\}$, and the production set P is:

$$S_2 \to ccS_2a \mid dS_2bb \mid \lambda$$

(a) Write 6 distinct strings that belong to L_1 but do not belong to L_2 (belong to $L_1 \setminus L_2$). If such strings do not exist, state it and explain why.

Answer:

cdd
cbb
cd
ba
bba
bbba

(b) Write 6 distinct strings that belong to L_2 but do not belong to L_1 (belong to $L_2 \setminus L_1$). If such strings do not exist, state it and explain why.

Answer:

dccabb
ddccabbbb
dddccabbbbbb
dccccaabb
dccccccaaabb
dccccccccaaabb

(c) Write 6 distinct strings that belong to L_1 and L_2 (belong to $L_1 \cap L_2$). If such strings do not exist, state it and explain why.

Answer:

ccdbba
ccddbbbba
cca
dbb
λ
ccccdbbaa

(d) Write 6 distinct strings over alphabet $\{a, b, c, d\}$ that do not belong to L_1 and do not belong to L_2 (belong to $\overline{L_1} \cap \overline{L_2}$). If such strings do not exist, state it and explain why.

Answer:

ac
bd
dc
acac
bdc
dca

(e) Write 6 distinct strings that belong to L_2^* but do not belong to L_2. If such strings do not exist, state it and explain why.

Answer:

ccacca
dbbdbb
ccdbbaccdbba
ccddbbbbadbb
ccddbbbbacca
ddbbbbcca

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Problem 2 Let L_1 be the language represented by the regular expression:

$$c^*d^*b^*a^*$$

Let L_2 be the language generated by the context-free grammar $G_2 = (V, \Sigma, P, S_2)$, where $\Sigma = \{a, b, c, d\}$, $V = \{S_2\}$, and the production set P is:

$$S_2 \to ccS_2a \mid dS_2bb \mid \lambda$$

(a) Write 6 distinct strings that belong to L_1 but do not belong to L_2 (belong to $L_1 \setminus L_2$). If such strings do not exist, state it and explain why.

Answer:

cc aa
dd bb
ccdd bbaa
ca
db
cb

(b) Write 6 distinct strings that belong to L_2 but do not belong to L_1 (belong to $L_2 \setminus L_1$). If such strings do not exist, state it and explain why.

Answer:

d ccabb
ddccabbbb
dcccc aabb
ccdccabba
cc dcc dbbabb a
cccc d ccabb aa

(c) Write 6 distinct strings that belong to L_1 and L_2 (belong to $L_1 \cap L_2$). If such strings do not exist, state it and explain why.

Answer:

ccdbba.
λ
cca
dbb
cccc aa
dd bbbb

(d) Write 6 distinct strings over alphabet $\{a, b, c, d\}$ that do not belong to L_1 and do not belong to L_2 (belong to $\overline{L_1} \cap \overline{L_2}$). If such strings do not exist, state it and explain why.

Answer:

ac
ab
ad
bd
bc
dc

(e) Write 6 distinct strings that belong to L_2^* but do not belong to L_2. If such strings do not exist, state it and explain why.

Answer:

cc acca
dbbdbb
ccaccacca
dbbcca dbb
ccdbba ccdbba
dccabb dbb

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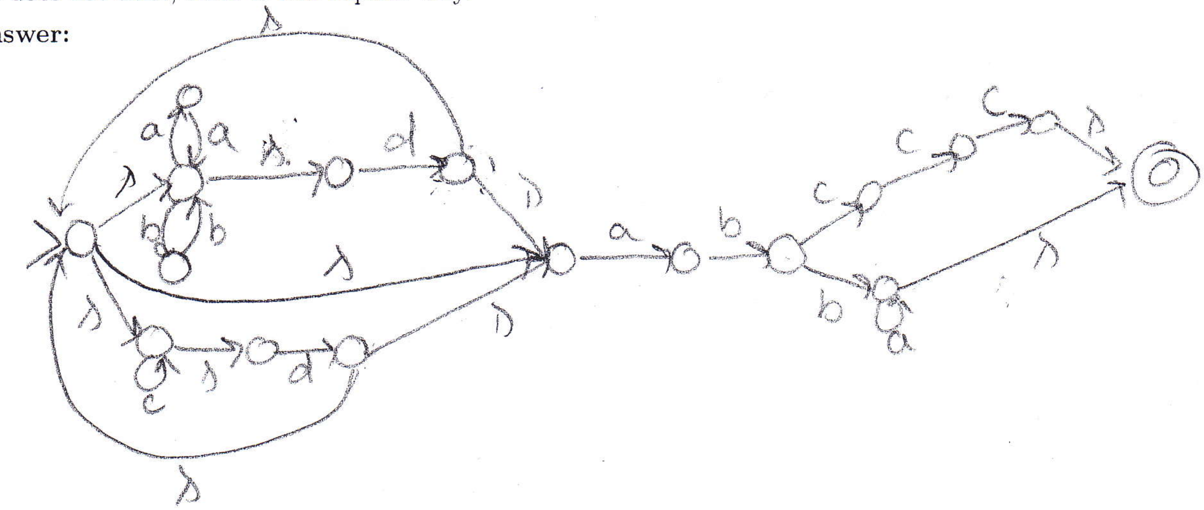
Problem 3 Let L be the language defined by the regular expression:

$$((aa \cup bb)^* d \cup c^* d)^* \; ab \; (ccc \cup ba^*)$$

(a) Draw a state-transition graph of a finite automaton that accepts the language L. If such an automaton does not exist, state it and explain why.

Answer:



(b) Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, S, P)$

$\Sigma = \{a, b, c, d\}$

$V = \{S, A, J, K, B, T\}$

P:

$S \to A\,ab\,B$

$A \to AA \mid Jd \mid Kd \mid \lambda$

$J \to JJ \mid aa \mid bb \mid \lambda$

$K \to KK \mid c \mid \lambda$

$B \to ccc \mid bT$

$T \to aT \mid \lambda$

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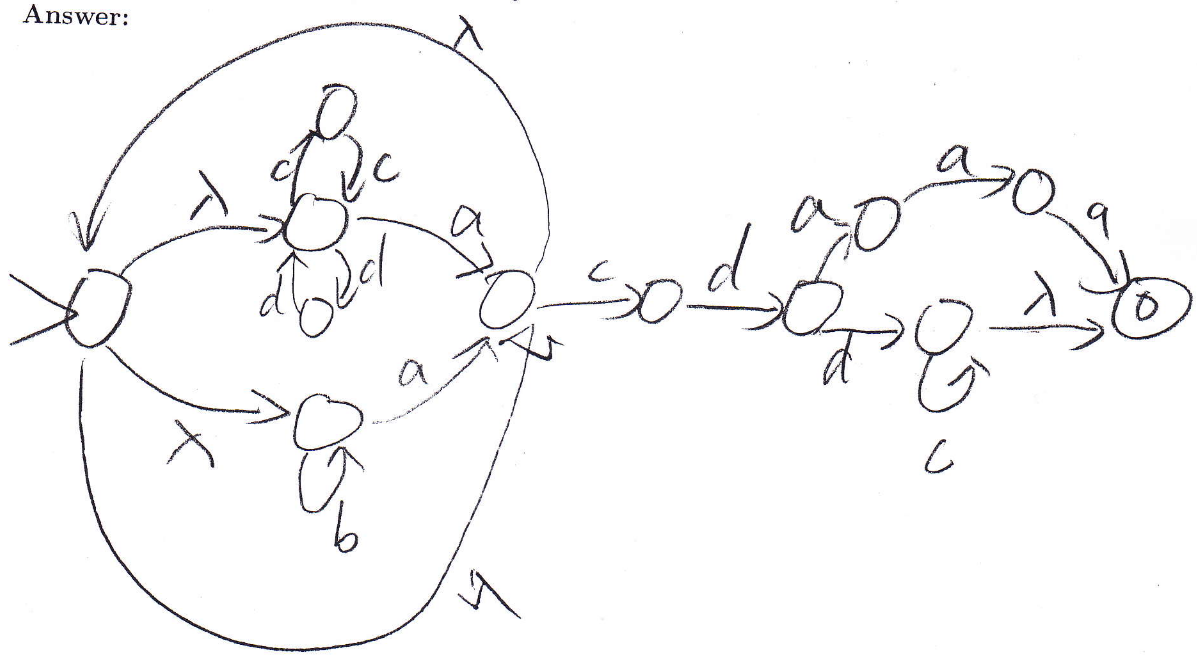
Problem 3 Let L be the language defined by the regular expression:

$$((cc \cup dd)^* a \cup b^* a)^* \; cd \; (aaa \cup dc^*)$$

(a) Draw a state-transition graph of a finite automaton that accepts the language L. If such an automaton does not exist, state it and explain why.

Answer:



(b) Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, state it and explain why.

Answer:

$G = \{V, \Sigma, P, S\}$, $\Sigma = \{a, b, c, d\}$

$V = \{S, A, B, C_1, H, F\}$

P: $S \to AcdB$

$A \to Ha \mid Fa \mid AA \mid \lambda$

$H \to ccH \mid ddH \mid \lambda$

$F \to bF \mid \lambda$

$B \to aaa \mid dC_1$

$C_1 \to C_1 c \mid \lambda$

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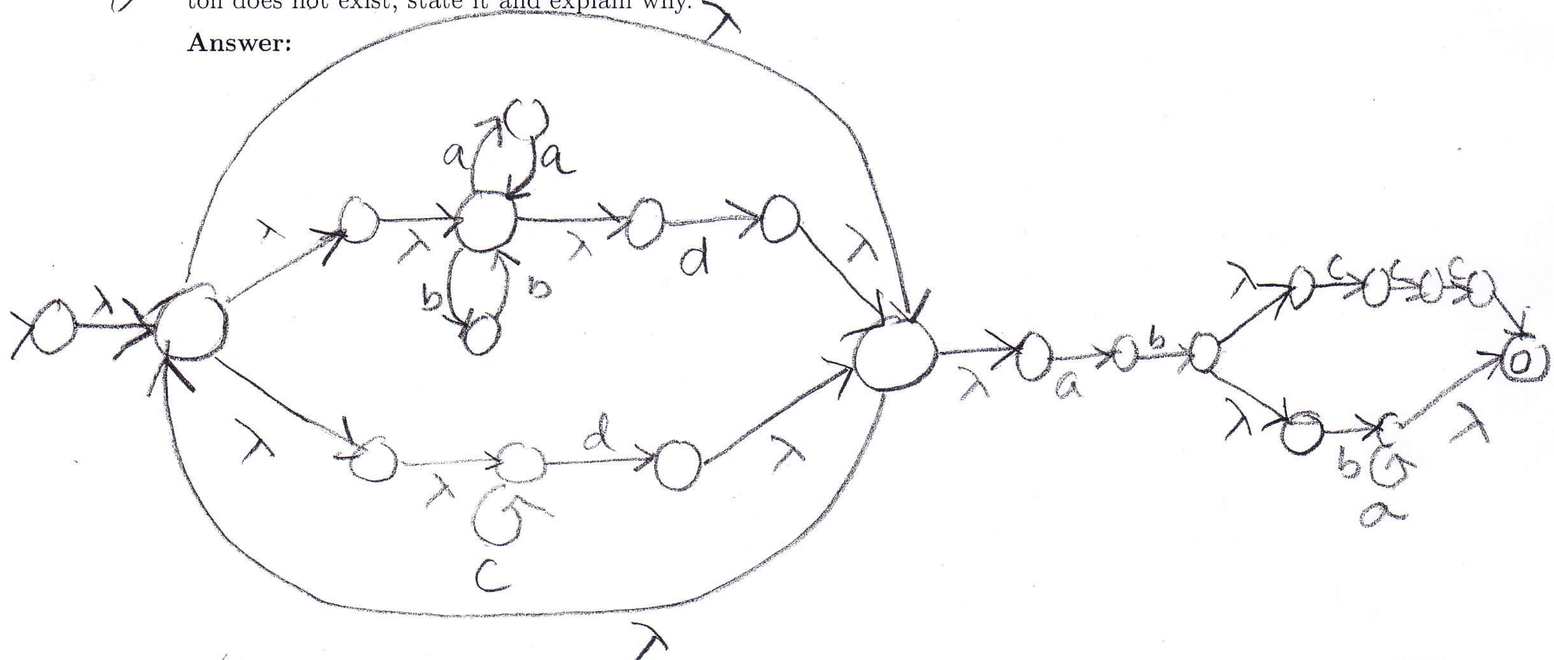
Problem 3 Let L be the language defined by the regular expression:

$$((aa \cup bb)^*d \cup c^*d)^* \; ab \; (ccc \cup ba^*)$$

(a) Draw a state-transition graph of a finite automaton that accepts the language L. If such an automaton does not exist, state it and explain why.

Answer:



(b) Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, state it and explain why.

Answer: $G = (V, \Sigma, S, P)$ $\Sigma = \{a, b, c, d\}$

$V = \{S, A, D, E, B, M\}$

P:

$S \to AabB$

$A \to AA \mid Dd \mid Ed \mid \lambda$

$D \to DD \mid aa \mid bb \mid \lambda$

$E \to EE \mid c \mid \lambda$

$B \to ccc \mid bM$

$M \to MM \mid a \mid \lambda$

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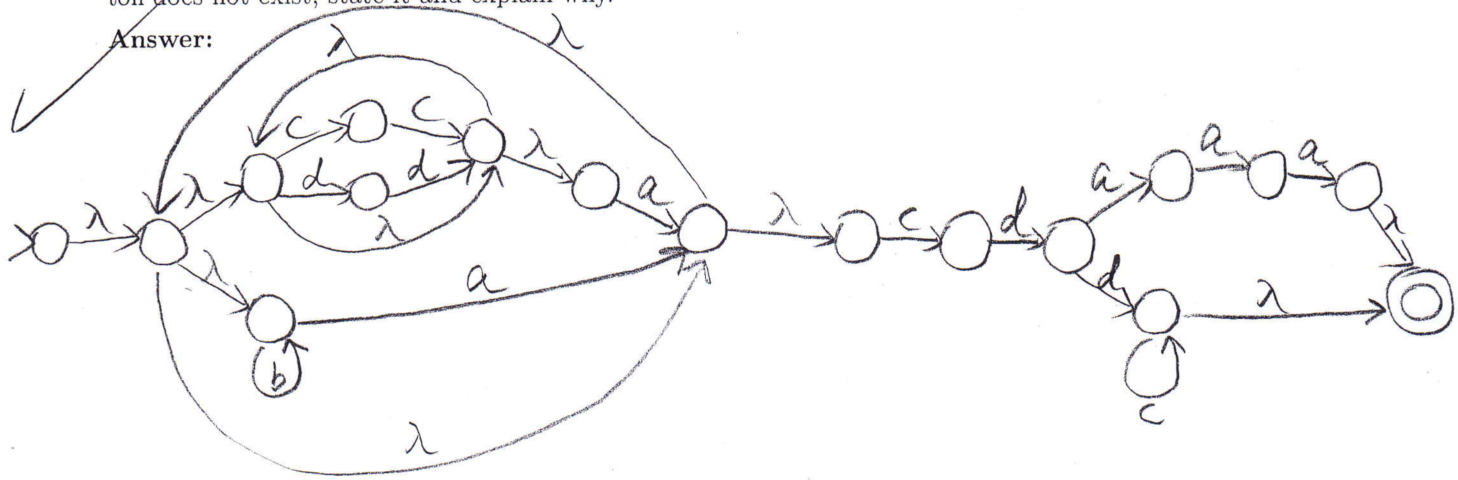
Problem 3 Let L be the language defined by the regular expression:

$$((cc \cup dd)^* a \cup b^* a)^* \; cd \; (aaa \cup dc^*)$$

(a) Draw a state-transition graph of a finite automaton that accepts the language L. If such an automaton does not exist, state it and explain why.

Answer:



(b) Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c, d\}$

$V = \{S, A, B, D, E, F, H\}$

P: $S \to AcdB$

$A \to \lambda \mid D \mid E \mid AA$

$D \to Fa$

$F \to \lambda \mid cc \mid dd \mid FF$

$E \to a \mid bE$

$B \to aaa \mid dH$

$H \to \lambda \mid cH$

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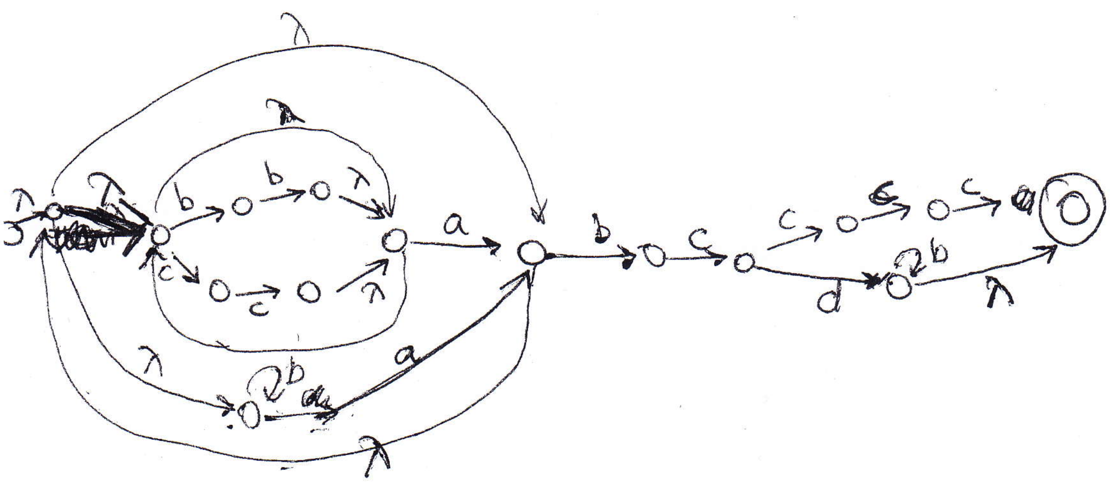
Problem 3 Let L be the language defined by the regular expression:

$$((bb \cup cc)^* a \cup b^* a)^* \; bc \; (ccc \cup db^*)$$

(a) Draw a state-transition graph of a finite automaton that accepts the language L. If such an automaton does not exist, state it and explain why.

Answer:



(b) Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c, d\}$

P: $V = \{S, A, B, D, E, K\}$

$S \to A\,bc\,B$

$A \to Da \mid Ea \mid \lambda \mid AA$

$D \to bb \mid cc \mid \lambda \mid DD$

$E \to \lambda \mid bE$

$B \to ccc \mid dK$

$K \to \lambda \mid bK$

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Problem 4 Let L_1 be a language over the alphabet $\{a, b, c, d, e\}$, defined as follows:

$$L_1 = \{a^m\, d^{2m}\, e^{3\ell}\, c^{4\ell}\, b^{5j} \text{ where } m, \ell, j \geq 0\}$$

Let L_2 be a language over the alphabet $\{a, b, c, d, e\}$, defined as follows:

$$L_2 = \{b^{m+1}\, c^{\ell+2}\, d^{j+3}\, a^{j+4}\, e^{\ell+5} \text{ where } m, \ell, j \geq 0\}$$

(a) Write a complete formal definition of a context-free grammar that generates L_1. If such a grammar does not exist, state it and explain why.

Answer:

$G_1 = (V_1, \Sigma, P_1, S_1)$

$\Sigma = \{a, b, c, d, e\}$

$V_1 = \{S_1, A, B, D\}$

P_1: $S_1 \to ABD$

$A \to \lambda \mid aAdd$

$B \to \lambda \mid eeeBcccc$

$D \to \lambda \mid bbbbbD$

(b) Write a complete formal definition of a context-free grammar that generates L_2. If such a grammar does not exist, state it and explain why.

Answer:

$G_2 = (V_2, \Sigma, P_2, S_2)$

$\Sigma = \{a, b, c, d, e\}$

$V_2 = \{S_2, E, F, H\}$

P_2: $S_2 \to EF$

$E \to b \mid bE$

$F \to ccHeeeee \mid cFe$

$H \to dddaaaa \mid dHa$

(c) Write a complete formal definition of a context-free grammar that generates $L_1 L_2$. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c, d, e\}$

$V = \{S_1, A, B, D, S_2, E, F, H, S\}$

P: $S \to S_1 S_2$

$S_1 \to ABD$

$A \to \lambda \mid aAdd$

$B \to \lambda \mid eeeBcccc$

$D \to \lambda \mid bbbbbD$

$S_2 \to EF$

$E \to b \mid bE$

$F \to ccHeeeee \mid cFe$

$H \to dddaaaa \mid dHa$

(d) Write a complete formal definition of a context-free grammar that generates L_1^*. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c, d, e\}$

$V = \{S_1, A, B, D, S\}$

P: $S \to \lambda \mid S_1 \mid SS$

$S_1 \to ABD$

$A \to \lambda \mid aAdd$

$B \to \lambda \mid eeeBcccc$

$D \to \lambda \mid bbbbbD$

Problem 4 Let L_1 be a language over the alphabet $\{a, b, c, d, e\}$, defined as follows:

LAST NAME:

FIRST NAME:

$$L_1 = \{a^m\, d^{2\ell}\, e^{3\ell}\, c^{4j}\, b^{5j} \text{ where } m, \ell, j \geq 0\ \}$$

Let L_2 be a language over the alphabet $\{a, b, c, d, e\}$, defined as follows:

$$L_2 = \{b^{m+1}\, c^{\ell+2}\, d^{\ell+3}\, a^{m+4}\, e^{j+5} \text{ where } m, \ell, j \geq 0\}$$

(a) Write a complete formal definition of a context-free grammar that generates L_1. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S_1)$ $\Sigma = \{a, b, c, d, e\}$

$V = \{S_1, A, B, D\}$

$S_1 \to ABD$

$A \to \lambda \mid aA$

$B \to \lambda \mid ddBeee$

$D \to \lambda \mid ccccDbbbbb$

(b) Write a complete formal definition of a context-free grammar that generates L_2. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S_2)$ $\Sigma = \{a, b, c, d, e\}$

$V = \{S_2, E, F, G\}$

$S_2 \to EF$

$F \to eeeee \mid eF$

$E \to bGaaaa \mid bEa$

$G \to ccddd \mid cGd$

(c) Write a complete formal definition of a context-free grammar that generates L_1^*. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S_3)$ $\Sigma = \{a, b, c, d, e\}$

$V = \{S_3, S_1, A, B, D\}$

$S_3 \to \lambda \mid S_1 S_3$

$S_1 \to ABD$

$A \to \lambda \mid aA$

$B \to \lambda \mid ddBeee$

$D \to \lambda \mid ccccDbbbbb$

(d) Write a complete formal definition of a context-free grammar that generates L_1L_2. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S_4)$ $\Sigma = \{a, b, c, d, e\}$

$V = \{S_4, S_1, S_2, A, B, D, E, F, G\}$

$S_4 \to S_1 S_2$

$S_1 \to ABD$

$A \to \lambda \mid aA$

$B \to \lambda \mid ddBeee$

$D \to \lambda \mid ccccDbbbbb$

$S_2 \to EF$

$F \to eeeee \mid eF$

$E \to bGaaaa \mid bEa$

$G \to ccddd \mid cGd$

LAST NAME:

FIRST NAME:

Problem 4 Let L_1 be a language over the alphabet $\{a, b, c, d, e\}$, defined as follows:

$$L_1 = \{\underbrace{a^m}_{E} \underbrace{d^{2\ell} e^{3\ell}}_{A} \underbrace{c^{4j} b^{5j}}_{B} \text{ where } m, \ell, j \geq 0\}$$

Let L_2 be a language over the alphabet $\{a, b, c, d, e\}$, defined as follows:

$$L_2 = \{\underbrace{b^{m+1} \underbrace{c^{\ell+2} d^{\ell+3}}_{F} a^{m+4}}_{D} \underbrace{e^{j+5}}_{Z} \text{ where } m, \ell, j \geq 0\}$$

(a) Write a complete formal definition of a context-free grammar that generates L_1. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V_1, \Sigma_1, P_1, S_1)$

$\Sigma = \{a, b, c, d, e\}$

$V = \{S_1, A, B, E\}$

P:

$S_1 \to EAB$

$A \to ddAeee \mid \lambda$

$B \to ccccBbbbbb \mid \lambda$

$E \to aE \mid \lambda$

(b) Write a complete formal definition of a context-free grammar that generates L_2. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V_2, \Sigma, P_2, S_2)$

$\Sigma = \{a, b, c, d, e\}$

$V = \{S_2, \cancel{A}, D, F, Z\}$

P:

~~$S_2 \to AB$~~

~~$A \to bAa \mid bEaaaa$~~

~~$E \to cEd \mid ccddd$~~

~~$B \to eB \mid eeeee$~~

$S_2 \to DZ$

$D \to bDa \mid bFaaaa$

$F \to cFd \mid ccddd$

$Z \to eZ \mid eeeee$

(c) Write a complete formal definition of a context-free grammar that generates L_1^*. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c, d, e\}$

$V = \{S, S_1, A, B, E\}$

P:

$S \to \lambda \mid SS \mid S_1$

$S_1 \to EAB$

$A \to ddAeee \mid \lambda$

$B \to ccccBbbbbb \mid \lambda$

$E \to aE \mid \lambda$

(d) Write a complete formal definition of a context-free grammar that generates $L_1 L_2$. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c, d, e\}$

$V = \{S, S_1, S_2, E, A, B, D, Z, F\}$

P:

$S \to S_1 S_2$

$S_1 \to EAB$

$S_2 \to DZ$

$A \to ddAeee \mid \lambda$

$B \to ccccBbbbbb \mid \lambda$

$E \to aE \mid \lambda$

$D \to bDa \mid bFaaaa$

$F \to cFd \mid ccddd$

$Z \to eZ \mid eeeee$

LAST NAME:

FIRST NAME:

Problem 4 Let L_1 be a language over the alphabet $\{a, b, c, d, e\}$, defined as follows:

$$L_1 = \{a^m\, d^{2\ell}\, e^{3\ell}\, c^{4j}\, b^{5j} \text{ where } m, \ell, j \geq 0\}$$

Let L_2 be a language over the alphabet $\{a, b, c, d, e\}$, defined as follows:

$$L_2 = \{b^{m+1}\, c^{\ell+2}\, d^{\ell+3}\, a^{m+4}\, e^{j+5} \text{ where } m, \ell, j \geq 0\}$$

(a) Write a complete formal definition of a context-free grammar that generates L_1. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S)$

$V = \{S, A, B, D\}$

$\Sigma = (a, b, c, d, e)$

P:

$S \to ABD$

$A \to AA \mid a \mid \lambda$

$B \to ddBeee \mid \lambda$

$D \to ccccDbbbbb \mid \lambda$

(b) Write a complete formal definition of a context-free grammar that generates L_2. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S)$

$V = \{S, F, H, E\}$

$\Sigma = \{a, b, c, d, e\}$

P:

$S \to FE$

$F \to bFa \mid bHaaaa$

$H \to cHd \mid ccddd$

$E \to eE \mid eeeee$

(c) Write a complete formal definition of a context-free grammar that generates L_1^*. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c, d, e\}$

$V = \{S, S_1, A, B, D\}$

P:

$S \to SS \mid S_1 \mid \lambda$

$S_1 \to ABD$

$A \to AA \mid a \mid \lambda$

$B \to ddBeee \mid \lambda$

$D \to ccccDbbbbb \mid \lambda$

(d) Write a complete formal definition of a context-free grammar that generates $L_1 L_2$. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c, d, e\}$

$V = \{S, S_1, S_2, A, B, D, F, H, E\}$

P:

$S \to S_1 S_2$

$S_1 \to ABD$

$A \to AA \mid a \mid \lambda$

$B \to ddBeee \mid \lambda$

$D \to ccccDbbbbb \mid \lambda$

$S_2 \to FE$

$F \to bFa \mid bHaaaa$

$H \to cHd \mid ccddd$

$E \to eE \mid eeeee$

Problem 4 Let L_1 be a language over the alphabet $\{a, b, c, d, e\}$, defined as follows:

LAST NAME:

FIRST NAME:

$$L_1 = \{\underbrace{a^{5m} d^{4m}}_{A} \underbrace{e^{3\ell} c^{2\ell}}_{B} \underbrace{b^j}_{D} \text{ where } m, \ell, j \geq 0\}$$

Let L_2 be a language over the alphabet $\{a, b, c, d, e\}$, defined as follows:

$$L_2 = \{\underbrace{b^{m+5}}_{A} \underbrace{c^{\ell+4} \underbrace{d^{j+3} a^{j+2}}_{D} e^{\ell+1}}_{B} \text{ where } m, \ell, j \geq 0\}$$

(a) Write a complete formal definition of a context-free grammar that generates L_1. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S)$ $V = \{S, A, B, D\}$

$\Sigma = \{a, b, c, d, e\}$

$P: S \to ABD$

$A \to aaaaaAdddd \mid \lambda$

$B \to eeeBcc \mid \lambda$

$D \to DD \mid b \mid \lambda$

(b) Write a complete formal definition of a context-free grammar that generates L_2. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S)$ $\Sigma = \{a, b, c, d, e\}$

$V = \{S, A, B, D\}$

$P: S \to AB$

$A \to bA \mid bbbbb$

$B \to cBe \mid ccccDe$

$D \to dDa \mid ddd\lambda$ No

$\boxed{aa}$

(c) Write a complete formal definition of a context-free grammar that generates L_2L_1. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S)$ $V = \{S, E, F, G, H, J, K, L, M\}$

$\Sigma = \{a, b, c, d, e\}$

$P: S \to FE$

$E \to GHJ$

$G \to aaaaaGdddd \mid \lambda$

$H \to eeeHcc \mid \lambda$

$J \to JJ \mid b \mid \lambda$

$F \to KL$

$K \to bK \mid bbbbb$

$L \to cLe \mid ccccMe$ No.

$M \to dMa \mid ddd\lambda$ $\boxed{aa}$

(d) Write a complete formal definition of a context-free grammar that generates L_2^*. If such a grammar does not exist, state it and explain why.

Answer: $G = (V, \Sigma, P, S)$ $V = \{S, A, B, D\}$

$S \to AB \mid SS \mid \lambda$ $\Sigma = \{a, b, c, d, e\}$

$A \to bA \mid bbbbb$

$B \to cBe \mid ccccDe$ No.

$D \to dDa \mid ddd\lambda$

$\boxed{aa}$

Problem 5 Let L be the set of all strings over the alphabet $\{a, b\}$ whose length is divisible by 2 or 9.

(a) Write a regular expression that represents the language L. If such a regular expression does not exist, state it and explain why.

Answer:

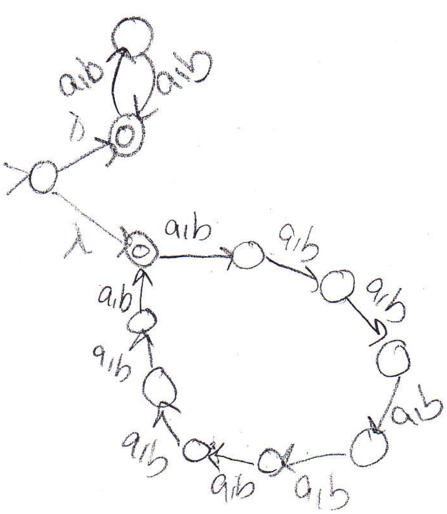
$((a \cup b)(a \cup b))^*$

$\cup$

$((a\cup b)(a\cup b)(a\cup b)(a\cup b)(a\cup b)(a\cup b)(a\cup b)(a\cup b)(a\cup b))^*$

(b) Draw a state-transition graph of a finite automaton that accepts the language L. If such an automaton does not exist, state it and explain why.

Answer:



LAST NAME:

FIRST NAME:

(c) Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, state it and explain why.

Answer:

$G = \{V, \Sigma, S, P\}$ $\Sigma = \{a, b\}$

$V = \{S, A, B, Z\}$

P:

$S \Rightarrow A \mid B$

$A \Rightarrow ZZA \mid \lambda$

$B \Rightarrow ZZZZZZZZZB \mid \lambda$

$Z \to a \mid b$

LAST NAME:

FIRST NAME:

Problem 5 Let L be the set of all strings over the alphabet $\{a, b\}$ whose length is divisible by 2 or 9.

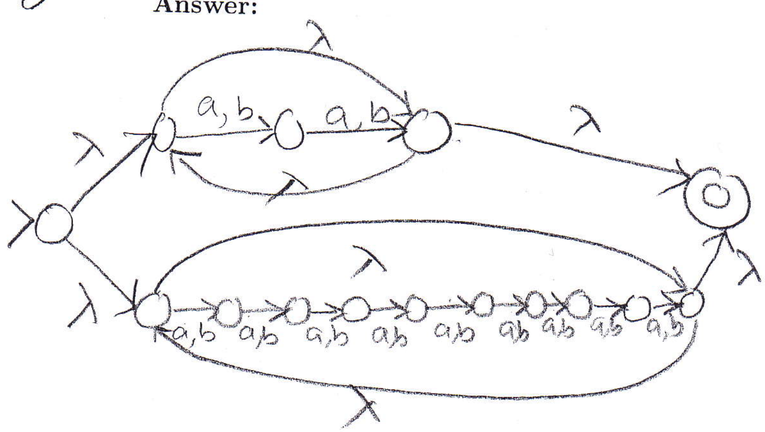
(a) Write a regular expression that represents the language L. If such a regular expression does not exist, state it and explain why.

Answer:

$$((a \cup b)(a \cup b))^* \cup ((a \cup b)(a \cup b)(a \cup b)(a \cup b)(a \cup b)(a \cup b)(a \cup b)(a \cup b)(a \cup b))^*$$

(b) Draw a state-transition graph of a finite automaton that accepts the language L. If such an automaton does not exist, state it and explain why.

Answer:



(c) Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, state it and explain why.

Answer: $G = (V, \Sigma, P, S)$

$\Sigma = \{a, b\}$ $V = \{S, A, B, Z\}$

P:

$S \to A \mid B$

$A \to AA \mid ZZ \mid \lambda$

$B \to BB \mid ZZZZZZZZZ \mid \lambda$

$Z \to a \mid b$

LAST NAME:

FIRST NAME:

Problem 5 Let L be the set of all strings over the alphabet $\{a, b\}$ whose length is divisible by 2 or 9.

(a) Write a regulãr expression that represents the language L. If such a regular expression does not exist, state it and explain why.

Answer:

$((a\cup b)(a\cup b))^* \cup$

$((a\cup b)(a\cup b)(a\cup b)(a\cup b)(a\cup b)(a\cup b)(a\cup b)(a\cup b)(a\cup b))^*$

(c) Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, state it and explain why.

Answer:

$G=\{V,\Sigma,P,S\}$

$V=\{S,A,B,D\}$

$S\to A|B$

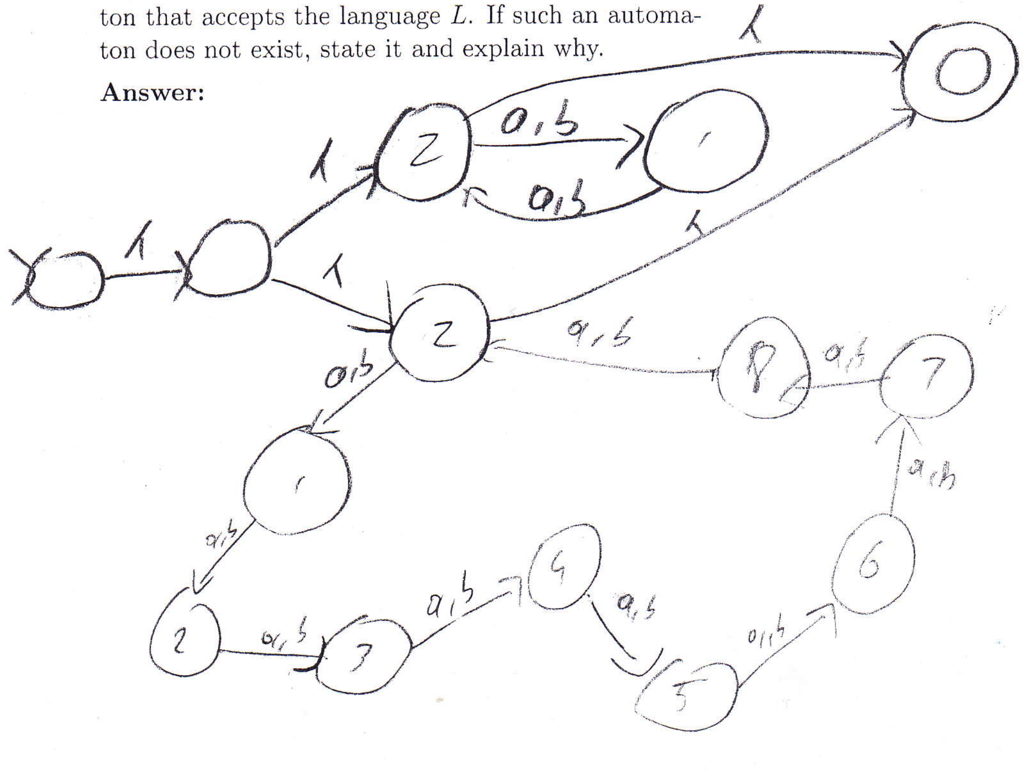
$A\to AA|DD|\lambda$

$D\to a|b$

$B\to BB|DDDDDDDDD|\lambda$

(b) Draw a state-transition graph of a finite automaton that accepts the language L. If such an automaton does not exist, state it and explain why.

Answer:



LAST NAME:

FIRST NAME:

Problem 5 Let L be the set of all strings over the alphabet $\{a, b\}$ whose length is divisible by 3 or 8.

(a) Write a regular expression that represents the language L. If such a regular expression does not exist, state it and explain why.

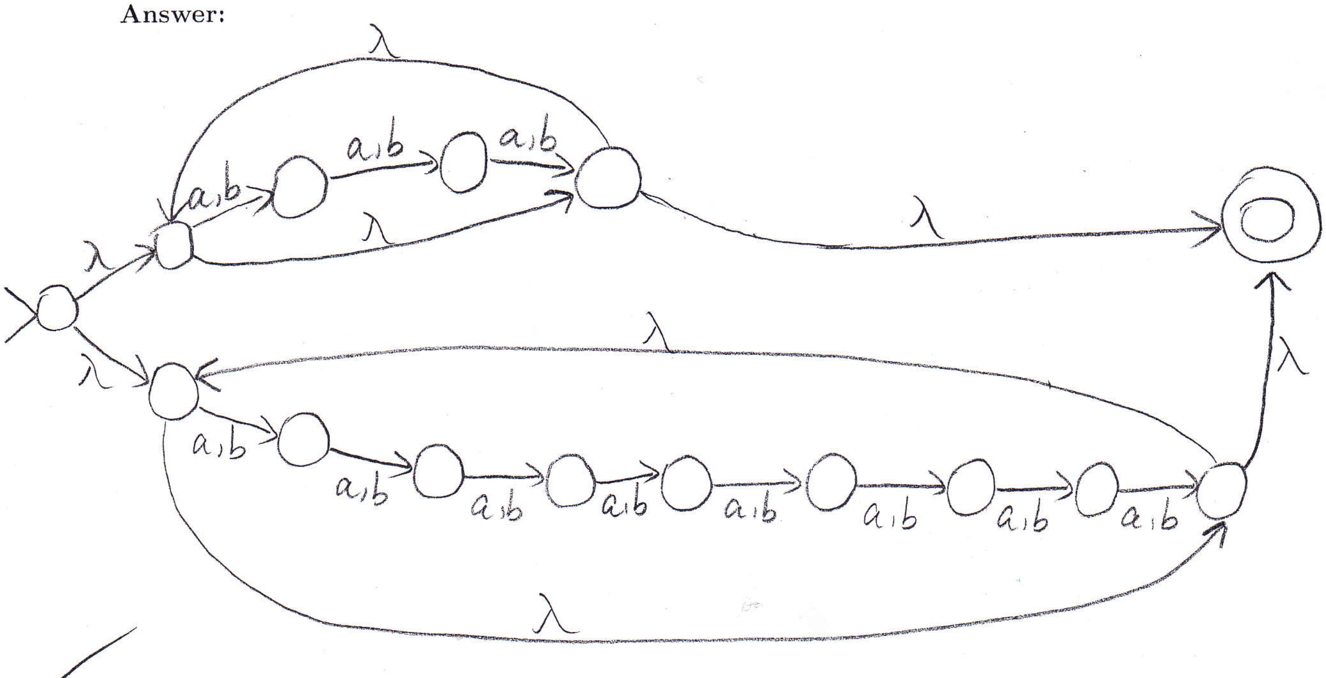
Answer:

Let $Z = (a \cup b)$, then the regular expression that represents L is:

$(ZZZ)^* \cup (ZZZZZZZZ)^*$

(b) Draw a state-transition graph of a finite automaton that accepts the language L. If such an automaton does not exist, state it and explain why.

Answer:



(c) Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b\}$

$V = \{S, S_1, S_2, Z\}$

P: $S \to S_1 \mid S_2$

$Z \to a \mid b$

$S_1 \to \lambda \mid ZZZ \mid S_1 S_1$

$S_2 \to \lambda \mid ZZZZZZZZ \mid S_2 S_2$

LAST NAME:

FIRST NAME:

Problem 6 Let L be the set of all strings over the alphabet $\{a, b, c\}$ which satisfy all of the following conditions:

1. contains an even number of c's;
2. contains at least one a.

(a) Write 5 distinct strings that belong to L. If such strings do not exist, state it and explain why.

Answer:

a
ab
abb
acc
abcc

(b) Write a regular expression that represents the language L. If such a regular expression does not exist, state it and explain why.

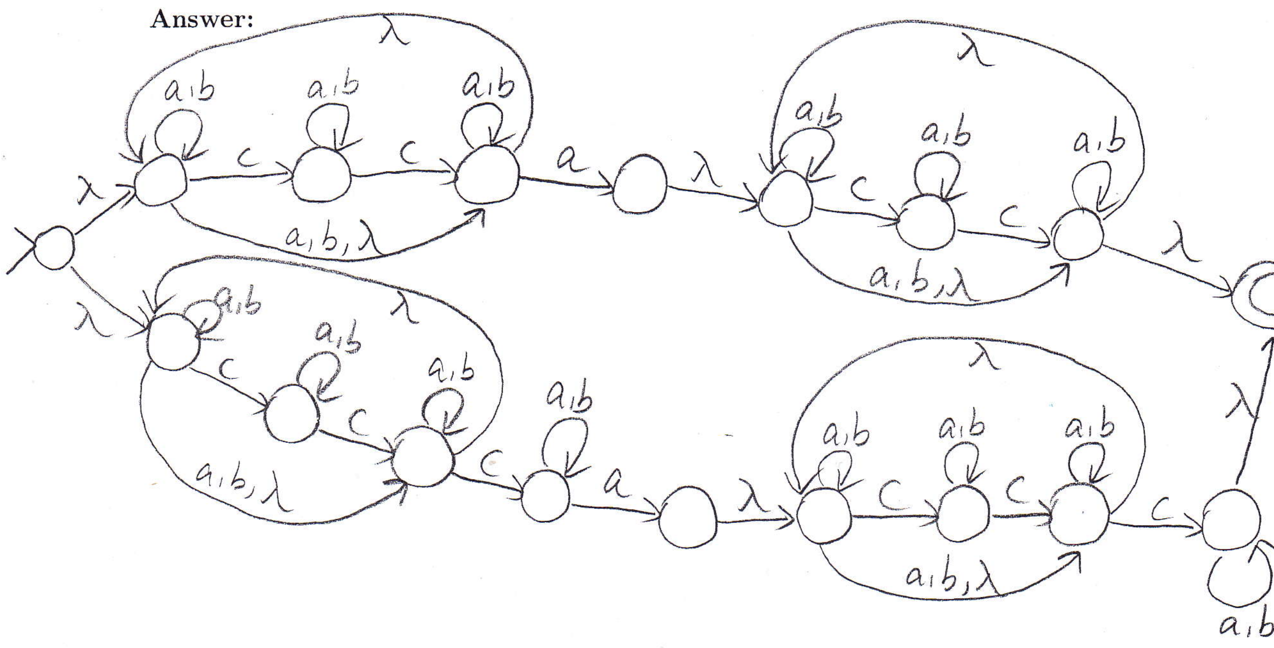
Answer:

Let $Z = (a \cup b)^*$, then the regular expression that represents L is:

$$(ZcZcZ)^* Z a (ZcZcZ)^* Z$$
$$\cup$$
$$(ZcZcZ)^* ZcZa(ZcZcZ)^* ZcZ$$

(c) Draw a state-transition graph of a finite automaton that accepts the language L. If such an automaton does not exist, state it and explain why.

Answer:



(d) Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c\}$

$V = \{S, E, D, Z, A\}$

P: $S \to EaE \mid DaD$

$Z \to \lambda \mid a \mid b \mid ZZ$

$A \to \lambda \mid ZcZcZ \mid AA$

$E \to AZ$

$D \to AZcZ$

LAST NAME:

FIRST NAME: Solution

Problem 6 Let L be the set of all strings over the alphabet $\{a, b, c\}$ which satisfy all of the following conditions:

1. contains an even number of c's;
2. contains at least one b.

(a) Write 5 distinct strings that belong to L. If such strings do not exist, state it and explain why.

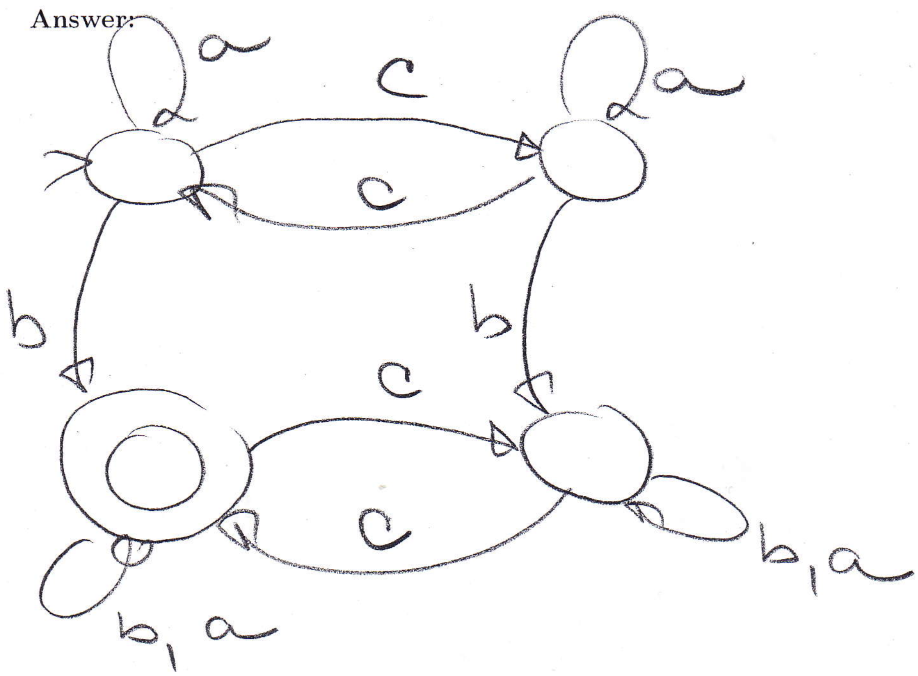
Answer:

(b) Write a regular expression that represents the language L. If such a regular expression does not exist, state it and explain why.

Answer:

(c) Draw a state-transition graph of a finite automaton that accepts the language L. If such an automaton does not exist, state it and explain why.

Answer:



(d) Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, state it and explain why.

Answer:

LAST NAME:

FIRST NAME:

Problem 7 Let L be the set of all strings over the alphabet $\{a, b, c\}$ which satisfy all of the following properties.

1. the length of the string is an odd number greater than 3;
2. the middle letter is not c;
3. the first letter is equal to the second letter;
4. the last letter is different from the next-to-last letter.

Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c\}$

$V = \{S, A, B, D\}$

P: $S \to BAD$

$A \to a \mid b \mid ZAZ$

$Z \to a \mid b \mid c$

$B \to aa \mid bb \mid cc$

$D \to ab \mid ac \mid ba \mid bc \mid ca \mid cb$

LAST NAME:

FIRST NAME:

Problem 7 Let L be the set of all strings over the alphabet $\{a, b, c\}$ which satisfy all of the following properties.

1. the length of the string is an odd number greater than 3;
2. the middle letter is not a;
3. the first letter is equal to the second letter;
4. the last letter is different from the next-to-last letter.

Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, state it and explain why.

Answer:

$G = \{V, \Sigma, S, P\}$ $\Sigma = \{a, b, c\}$

$V = \{S, A, Z, T, J, K\}$

P:

$S \to A$

$A \to TZJ$

$Z \to KZK \mid b \mid c$

$T \to aa \mid bb \mid cc$

$J \to ab \mid ac \mid bc \mid ba \mid cb \mid ca$

$K \to a \mid b \mid c$

LAST NAME:

FIRST NAME:

Problem 7 Let L be the set of all strings over the alphabet $\{a, b, c\}$ which satisfy all of the following properties.

1. the length of the string is an odd number greater than 3;
2. the middle letter is not a;
3. the first letter is different from the second letter;
4. the last letter is equal to the next-to-last letter.

Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, state it and explain why.

Answer:

$G = (V, \Sigma, S, P)$

$V = \{S, L, M, R, E$

$\Sigma = \{a, b, c\}$

P:

$S \to LMR$

$L \to ab \mid ac \mid bc \mid ba \mid ca \mid cb$

$M \to aMa \mid aMb \mid aMc \mid bMa \mid bMb \mid bMc \mid cMa \mid cMb \mid cMc \mid E$

$E \to b \mid c$

$R \to aa \mid bb \mid cc$

$\rightarrow$

"$=$" "$\neq$" "$\rightarrow$" "!"